Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Numerical Methods

Subject Code: 4SC04MTE1 **Branch: B. Sc. (Mathematics)**

Semester: IV **Date: 21/11/2015** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Which of the following is transcendental equation

1) $x - 2 = 0$ 2) $x^2 - 3x + 6 = 0$
3) $xe^x - 2 = 0$ 4) None of these

b) A real root of the equation $x^3 - x - 11 = 0$ lies in

1) $(0,1)$ 2) $(2,3)$
3) $(1,2)$ 4) None of these

c) The order of convergence in Bisection method is

1) linear 2) zero
3) quadratic 4) None of these

d) Which of the following is a step by step method:

1) Taylor's 2) Picard's
3) Euler's 4) None of these

e) **Match the following:**

A	Newton-Raphson	1	Integration
B	Runge-kutta	2	Root finding
C	Simpson's Rule	3	Ordinary Differential Equations

1) $A2 - B3 - C1$ 2) $A1 - B3 - C2$
3) $A2 - B1 - C3$ 4) None of these

f) The number of strips required in Weddle's rule is …………

1) 2   2) 4

3) 6   4) 8

g) Newton's iterative formula to find the value of $\sqrt{N}$ is

1) $x_{n+1} = \left(x_n + \frac{N}{x_n}\right)$ 2) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$

3) $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ 4) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$

h) In Regula-falsi method, the first approximation is given by………….. .

i) Using forward differences, the formula for $f'(a) =$………… .

j) Taylor's series solution of $y' = -xy, y(0) = 1$ up to $x^4$is ………... .

k) The second order Runge-Kutta formula is Modified Eular's method. Determine whether the statement is True or False.

l) In Eular's Method if h is small the method is too slow, if h is large, it gives inaccurate value. Determine whether the statement is True or False.

m) Whenever Trapezoidal rule is applicable, Simpson's 1/3rd rule can also be applied. Determine whether the statement is True or False.

n) Runge-kutta method is a self-starting method. Determine whether the statement is True or False.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Compute real root of $x^3 - 9x + 1 = 0$, correct to two decimal places, by bisection method. **(05)**

b) Find by Newton-Raphson Method the real root of $3x - \cos x - 1 = 0$, correct to five decimal places. **(05)**

c) Find $y(0.10)$ and $y(0.15)$, by Euler's Method, from the differential equation, $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$, correct up to four decimal places, taking step length $h = 0.05$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Let $x = \xi$ be a root of $f(x) = 0$ and let I be an interval containing the point $x = \xi$. Let $\phi(x)$ and $\phi'(x)$ be continuous in I where $\phi(x)$ is defined by the equation $x = \phi(x)$ which is equivalent to $f(x) = 0$. Then prove that if $|\phi'(x)| < 1$ for all x in I, the sequence of approximations $x_0, x_1, x_2, \dots, x_n$ defined by $x_n = \phi(x_{n-1})$ converges to the ξ, provided that the initial approximation x_0 is chosen in I. **(05)**

b) Compute one root of $e^x - 3x = 0$, correct to two decimal places which between 1 and 2. **(05)**

c) Compute $\sqrt{27}$ correct up to six decimal places. **(04)**

Q-4 **Attempt all questions** **(14)**

a) Use Picard's method to compute $y(0.1)$, from the differential equation $\frac{dy}{dx} = x + y\,; y(0) = 1$. **(05)**

b) Compute $y(0.6)$, by Runge-Kutta method correct to five decimal places, from the equation $\frac{dy}{dx} = xy,\ \ y(0) = 2$, taking $h = 0.2$. **(05)**

c) Evaluate $\int_0^1 x^3\ dx$, by Trapezoidal Rule, with $n = 5$. **(04)**

Q-5 **Attempt all questions** **(14)**

a) Derive differentiation formulae based on Newton's backward formula. **(05)**

b) Compute $f'(1.1)$ and $f''(1.1)$ from the following table **(05)**

x	1.1	1.2	1.3	1.4	1.5
$f(x)$	2.0091	2.0333	2.0692	2.1143	2.1667

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$, by using Weddle's rule. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Compute $y(0.5)$, by Milne's predictor corrector method from $\frac{dy}{dx} = 2e^x - y$ given that $y(0.1) = 2.0100, y(0.2) = 2.0401, y(0.3) = 2.0907, y(0.4) = 2.1621$ **(05)**

b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places. **(05)**

c) Apply Euler-Maclaurin sum formula to find the sum $1^3 + 2^3 + 3^3 + \cdots + n^3$. **(04)**

Q-7 **Attempt all questions** **(14)**

a) Derive Trapezoidal rule. **(10)**

b) Evaluate: $\int_{0.1}^{0.7} (e^x + 2x)\, dx$ by Simpson's One third rule, taking $h = 0.1$, correct to 5-decimal places. **(04)**

Q-8 **Attempt all questions** **(14)**

a) State and prove Euler-Maclaurin Sum Formula. **(05)**

b) Derive differentiation formulae based on Newton's divided difference formula. **(05)**

c) Describe Picard's Method for first order ordinary differential equation. **(04)**